Experimentally Determining Diffraction Grating Constants

REFERENCES:

Determination of the grating constants of the holographic grating using a He-Ne-laser. LD Didactic Physics Leaflets, P5.7.2.4.


EQUIPMENT: Red He:Neon Laser (632.8 nm), goniometer, rotating component holder, assorted diffraction gratings, spectrometer, mercury spectral tube, gas discharge power supply.

INTRODUCTION:

Due to the wave nature of light, a source passing through slits on a scale similar to its wavelength will produce an interference pattern. This pattern depends on the wavelengths of the source, the line density of the grating, and the orientation of the grating. In this experiment, we will explore these relations with a monochromatic laser source, as well as with the atomic spectrum emitted by a tube of incandescent hydrogen.

There are two separate experiments designed around the same concept: measuring the interference peaks for several different diffraction gratings. You will first be given a grating with a known line density, and then two “mystery gratings” which you will try to measure through this lab. By observing the angle of diffraction in these peaks you will be able to ascertain the line spacing of the grating. The first part involves using a helium-neon laser as the light source and

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**Figure 1:** Diffraction of light through a grating, showing the arrangement of interference peaks on a background screen.
the second utilizes an emission spectrum of mercury gas.

THEORY:

Light passing through a series of slits will result in interference peaks and troughs. Projected on a screen, a pattern of light and dark areas is produced. Including the central “zero-order” peak, there can be multiple orders of the same pattern, symmetrically on each side. This is ordinarily described by the equation

\[ n\lambda = dsin\theta \]

(1)

Equation 1 simply tells us that in order for the waves to constructively interfere at point P or P’ the phase difference \(dsin\theta\) must be equal to an integral number of the light wavelength.

(where \(n\) is the order, \(\lambda\) is the wavelength, \(d\) is the spacing between slits, and \(\theta\) is the angle of the pattern). However, if we rotate the angle of incidence of the grating, a more general equation is required. For this we instead employ

\[ n\lambda = d(sina \pm sin\beta) \]

(2)

Where \(d\) is the grating constant \((N = 1/[1000 \ast d])\), with \(N\) being number of grating lines per mm, \(\alpha\) is the angle of incidence (the angle which we will rotate the grating), and \(\beta\) is the diffraction angle (relative to \(\alpha\)). Exercise: Derive equation 2. For more information see page (34) of the AO Spencer spectrometer manual or page (17) of the Gaertner Peck Spectrometer manual. In many cases Eq. 1, and the set-up it describes, is the simplest and easiest method, but there are many cases where Eq. 2 must be used. For example, to resolve Sodium doublet splitting clearly with standard lab equipment, a 600 line/mm grating must be rotated approximately 10 degrees.

By measuring diffraction patterns at different angles and configurations based on this initial equation we will be able to determine the grating constant \(d\), and the line density of the gratings.

PART A: DIFFRACTION OF LASER LIGHT

PLAN OF THE EXPERIMENT:

In this experiment, we will pass a laser beam through diffraction gratings and observe the angle of the diffraction peaks. Our apparatus is very simple. A laser is aligned above the immobile arm of an angle measuring system. The grating, which may rotate, is centered, and the second arm,
which holds the target for the interference peaks, can move relative to it in order to find the angle of diffraction. We will first find the angle of diffraction by examining the angle for $\alpha$ at which the light is directed straight back at the laser, for several peaks. Then we will measure the angle of diffraction for different rotations of the grating.

**PROCEDURE:**

Be very careful never to look into the beam of the laser, and be aware of the fact that the laser light may be reflected off the gratings at an unexpected angle. For this reason, avoid any movements that put your eyes at the same height as the laser apparatus, be conscious of reflected light, and close the laser shutter when you are not taking measurements.

We will be rotating both the grating as well as the target, so the values observed will depend on both of these angles. For a Helium-Neon laser, the wavelength of light generated is known to be $\lambda = 632.8$ nm.

First you should try each of these measurements with the known grating, check if your result is approximately correct, and then attempt the same with the mystery gratings.

**Setup:**

1. Position the laser on its base at the end of the immobile arm, such that it is exactly aligned with the arm. Put the target at the other end on the swiveling arm.
2. Place the diffraction grating in its base, set perpendicular to the laser source.

First we will use the “Littrow setup,” in which we look at the angle at which light reflected off the grating may be directed straight back into the laser. In this orientation $\alpha=\beta$, and therefore the maxima are described by the equation

$$d = \frac{n\lambda}{2\sin\alpha}$$

(3)

1. You will read $\alpha$ by looking at the angles written on the base of the piece holding the grating. There is a notch that indicates the angle. In this lab, $\alpha=0$ is the reading when the grating is perpendicular to the laser beam.
2. Turn on the laser and rotate the grating until the first maximum is directed straight back at the source.
3. Do this for the both maxima to the left and right, so that you have at least two measurements of the $\alpha$ for each.
4. Repeat for higher orders if they are visible.
Figure 2: Angles $\alpha$, $\beta$, $\omega$, for the laser diffraction set-up, for light passing through the grating and on to a target. The angle measured from the target arm is $\omega$, and due to the similar angles, $\beta = \omega - \alpha$.

Now we will measure the angles of $\alpha$, $\beta$ for light passing through the grating. After setting a value of $\alpha$ by rotating the central grating, you will measure $\beta$ by moving the “Target,” a white card at the end of the movable arm, to a position where the laser light is in the exact center of the card.

1. Just as before, $\alpha$ is the angle by which you have rotated the base, but since we are now measuring $\beta$ as well, we need to account for this in getting our $\beta$ value. If omega is the angle of the Target arm (where $\omega=0$ is with the arm straight out, and the target aligned in a line from the laser to the grating to the target), then $\beta = \omega - \alpha$. (Note that $\beta$ may be a negative number).

2. With $\alpha=0$, measure $\beta$ values for as many orders as possible, then rotate the grating and repeat for at least 2 more $\alpha$ angles.

3. It may be helpful to do a calculation for $d$ occasionally as you go, in order to make sure that you are measuring the correct angles and getting reasonable results.
Example data: For a grating listed as N=600 lines/mm, a cursory measurement of a few data points gives:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta = \omega - \alpha$</th>
<th>N (lines/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>22°</td>
<td>593</td>
</tr>
<tr>
<td>30°</td>
<td>23° - 30° = -7°</td>
<td>598</td>
</tr>
<tr>
<td>40°</td>
<td>-15°</td>
<td>607</td>
</tr>
</tbody>
</table>

PART B: EXPERIMENTALLY DETERMINING GRATING CONSTANT BY DIFFRACTION OF HYDROGEN SPECTRUM

INTRODUCTION:

Emission spectra are produced by electrons in an atom stepping down to a lower energy state. Conservation of energy requires that the electron must give up an amount of energy equal to the difference between the two states, which it does by producing a photon (the frequency of which, in turn, depends on its energy). These values are fixed by laws of quantum mechanics and depend on the atom’s properties, so each spectral pattern is unique to a certain element.

This lab relies on the fact that light of different wavelengths will be diffracted at different angles, and thus uses the grating to split the emission lines like a prism.

PLAN OF THE EXPERIMENT:

In this part of the experiment we pass light produced by a spectral emission source through a grating and measure the angle of diffraction for the main visible spectral lines. Additionally, by rotating both the diffraction grating as well as the target, the values observed will depend on both angles, as well as the order of the interference peaks. In order to find the density of the line grating, we need to know the wavelength of light. In the previous experiment, the laser is a monochromatic source, in this case, the emission lines from the lamp are each of a known frequency.
Figure 3: Layout of spectroscopy experiment.

PROCEDURE:

The spectrometer uses a vernier scale for measurements. To read a vernier scale, first find where the zero point aligns with the angle on the table—if it is between two, the degree value is the lower point. The decimal value is given by the line (from 0-9) that matches the hash mark on the table.

As before, it is wise to start with the grating of known grating constant, and verifying your results before proceeding to the mystery gratings.

Figure 4: Vernier scale reading 182.7 degrees.
Setting up the spectrometer:
1. Place the emission source in front of the spectrometer and turn it on. Place the grating in its mount and be sure that it is held firmly in place.
2. Look into the eyepiece and align the telescope and collimator by moving the telescope arm until you see a bright central peak. Try shifting the light source very slightly until this zeroth order line is as bright as possible (even being slightly out of the way diminishes the brightness seen in the eyepiece a great deal). Focus the beam with the knobs on the sides of the collimator and telescope until the line is clearly resolved and the crosshair is visible.
3. Find the spectrometer’s zero-angle: The angle at which the grating is fully perpendicular to the collimated light (depending on your spectrometer, this may not be a round number). Make sure that the grating table can move freely (but not so loose that you may move it by accident while moving the eye piece), this is controlled by a knob under the table. With the eye piece still centered on the central light peak, rotate the table so that the mount is exactly perpendicular to the collimator. You may need a card or ruler to accomplish this. **Note this angle—all measurements are relative to this number.**
4. [Later in the experiment, for \( \alpha > 0 \) alignments, you will repeat Steps 2 & 3 (except for making the grating perpendicular to the collimator) to find or set \( \alpha \).]

Experimentally Determining the Grating Constant:

For the Mercury source, there should be several brightly visible lines:

- Violet 1: 404.7 nm
- Violet 2: 407.8 nm
- Blue: 435.8 nm
- Green: 546.1 nm
- Yellow 1: 577 nm
- Yellow 2: 579.1 nm

1. With the grating in the \( \alpha=0 \) position, measure the angles of as many diffraction peaks as possible. Depending on the grating, it may be possible to observe several orders. Attempt to do so, measuring from both the right and left angles. Note the changes in the appearance of peaks as the angle increases.
2. After observing the \( \alpha=0 \) lines, make measurements for at least 2 more \( \alpha \) positions. In order to change the orientation of the grating, first position the eye piece at theta-zero, with the central peak in focus. Rotate the table between 5-30 degrees, make sure that the telescope is still aligned, and note the new zero-angle. Measurements of \( \beta \) are now relative to this.
3. Observe the peaks of this new \( \alpha \) angle, again measuring as many peaks as you can see. Repeat 2-3 until you have measurements for at least one order of spectral lines for at least 2 values of \( \alpha \).
We now show some sample student data illustrating a calibration of a 600 line/mm diffraction grating. The student chose the angle alpha to be zero and plotted the wavelengths of 4 known mercury lines versus the spectrometer angle $\theta$. A least squares fit of the data using the Capstone plotting software gives $d = 1.643 \times 10^{-6} \pm 5.6 \times 10^{-9} m$. See figure 5.

**Figure 5: Graph of $\lambda$ versus $\sin \theta$ for 4 Mercury wavelengths**

If we now used our calibrated grating to measure the wavelengths of the Balmer series (experiment 420) of Hydrogen the uncertainty in our measured wavelengths can be shown to be:

$$\frac{\Delta \lambda}{\lambda} = \left[ \frac{(\Delta d)}{d}^2 + (\cot \theta \Delta \theta)^2 \right]^{1/2}$$

(4)

where $d$ and its uncertainty are determined by the methods discussed in this write up.
ANALYSIS OF DATA:

Your goal through these two experiments is to produce a measurement of the line spacing, N, in the mystery gratings, as well for the “known” grating (who’s given value, though close, may not be precisely correct). Do so, including calculations for the error of your measurements. Provide your average measurements of N for each $\alpha$ in each experiment, as well as overall.

DISCUSSION:

Some discussion points:

What are the sources of error in these experiments?

If some of your results deviate from your measured value, is there any trend in the deviation which depends on the angle $\alpha$?

You may have seen small, dimmer, points of light around the diffraction peaks in the laser experiment when using the blazed holographic grating. Where did these come from?

If we replaced the light sources (laser and hydrogen emission tube) with a simple flashlight, would it be possible to perform either experiment? Why or why not?