Quantum Oscillations

AKA

Electronic Properties of a Two-Dimensional Electron Gas

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Layout and procedure edited 4/15/18 by DGH

Purpose

The purpose of this lab is to observe the unique properties of a two-dimensional electron system. Semiconductors are physically interesting materials because they can act like either insulators or conductors based on small changes in their environments. The semiconductor investigated in this lab is a p-type silicon substrate which is part of a MOSFET (Metal Oxide Semiconductor Field Effect Transistor). When an external electric field is applied via the gate of the MOSFET (see Figure 1), electrons form a "two-dimensional electron gas" or 2DEG in the p-type silicon substrate. Further, when a magnetic field is applied to a 2DEG, electrons are forced into distinct energy levels called "Landau levels". In this lab, you will apply external electric and magnetic fields to the silicon substrate to create and manipulate a 2DEG. In doing so, you will draw conclusions about the silicon substrate's properties.

Theory

The MOSFET

A MOSFET is a common type of transistor. A typical cross-section is sketched in Figure 1.

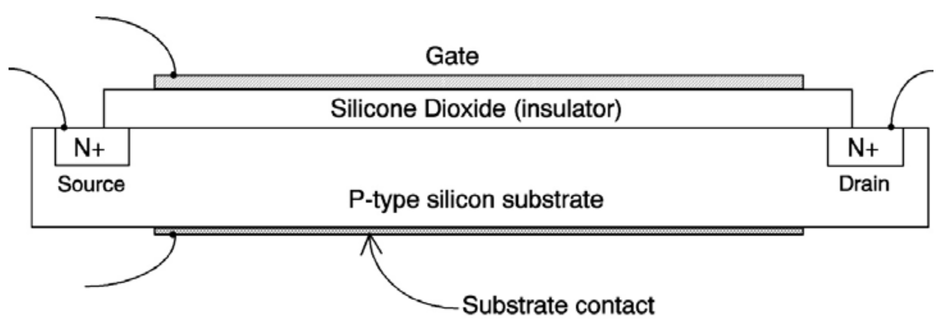


Figure 1: Cross-section of a typical MOSFET with p-type substrate.

A transistor is any device that selectively allows current flow. Transistors will only conduct when a sufficient potential is applied from an external source, and will otherwise act like an insulator. Transistors have several basic parts. The source and drain are the points through which current can flow, while the gate is where the external potential is applied. MOSFETs also have a fourth connection, the "body", which directly contacts the silicon substrate and can influence the gate potential required to make current flow.

In a MOSFET, the gate is a conducting sheet. It is insulated from the substrate and from the source and drain contacts by the oxide layer, so a potential difference can be created between the gate and the bottom of the substrate. The gate potential can induce a change in the ability of the semiconductor to conduct current between the source and drain. In silicon MOSFETs, the insulating layer is made of silicon dioxide, which has a dielectric constant of about 3.9 and is usually on the order of 100nm thick.

MOSFET samples generally have one of the three shapes in Figure 2:

Left:	A rectangle with contacts at each end only.
Center:	A rectangle with end contacts and pairs of side contacts, used for measuring the three independent components of the magneto-resistivity tensor.
Right:	A Corbino disk (with cylindrical symmetry) with interior and full exterior contacts, useful for measuring the longitudinal component of the conductivity tensor.

This lab will only use the geometry shown on the left, rectangular with two contacts.

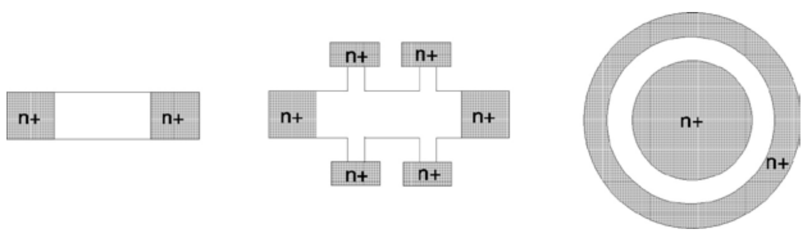


Figure 2: Geometry of samples

Semiconductors

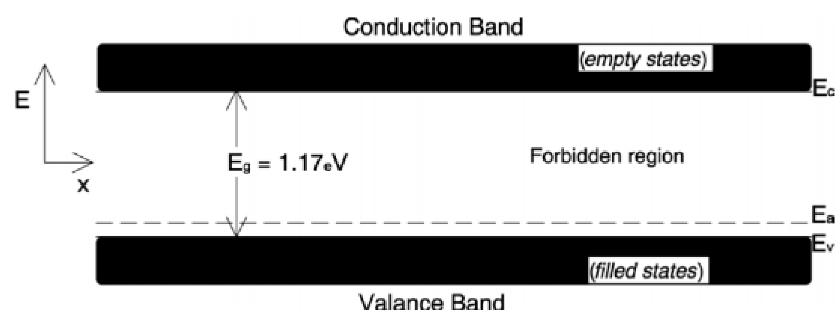
In many solid materials, electrons occupy energy levels known as "bands". All energy levels within each band can be occupied by electrons, while none outside the bands are accessible. This results in energy gaps in which certain energies are not accessible by electrons at all. In conducting materials, electrons can occupy a "conduction band" in which electrons are free to move and conduct through the material. In insulating materials, this conduction band is far above the Fermi energy of the material and inaccessible to the electrons.

In a semiconductor at $T = 0$, electrons completely fill their highest energy band (called the "valence band"). Additionally, there is a conduction band whose minimum energy is only slightly higher than the maximum energy of the valence band. The Fermi energy is located in the forbidden region between these two bands. When a potential is applied to the semiconductor, all existing electron states are forced up or down in energy. When a potential greater than a particular threshold potential V_T is applied, the minimum energy of the conduction band is reduced to less than the Fermi energy and conduction can occur. The threshold potential is determined by the difference in energy between the valence band and the conduction band, and this difference is called the "semiconducting energy gap."

In a pure or "intrinsic" semiconductor, electrons can only occupy states within the allowed bands of the material. However, many semiconducting materials are doped with impurities that reduce the energy required for electrons to enter the conduction band. These are "extrinsic semiconductors". There are two types of extrinsic semiconductors:

- "N-type" semiconductors are dominated by impurities called "donors." Donors carry electrons that can be "donated" to the semiconductor. These electrons are at an energy level well above the valence band and only slightly below the conduction band. Thus, electrons only need to overcome the energy gap between E_{donor} and $E_{conduction}$ to enter the conduction band. They are called "n-type" because it is electrons (negatively charged) that carry current.
- "P-type" semiconductors are dominated by "acceptors". Acceptors have empty energy states that can store electrons. These energy states are only slightly higher than the valence band. Therefore, they can "accept" electrons from the valence band that have a little extra energy. By removing these electrons from the valence band, acceptors create positively charged "holes" or empty energy states that would otherwise be occupied by electrons. It is these positive holes that carry the current (hence "p-type"). Electrons must only overcome the gap between $E_{valence}$ and $E_{acceptor}$ in a p-type semiconductor for conduction to occur.

The MOSFET used in this lab contains a p-type semiconductor. A typical p-type semiconductor is illustrated in Figure 3. For more on band theory and semiconductor fundamentals, see Solymar and Walsh [4] chapters 7 and 8.



3: A semiconductor energy diagram. The lower black band is the valence band, the upper black band is the conduction band, and the middle white band consists of forbidden states. In this example, the semiconductor is doped with an acceptor with acceptor level E_a between the valence and conduction bands, reducing the semiconductor's energy gap from E_g to E_a

The Formation of a 2DEG

When a sufficient potential is applied to a semiconductor, it acts like a conductor. However, it is a universal property of conductors that when a potential is applied, charges flow to the surface of the conductor to correct the potential. The result of this is that the surface of the semiconductor closest to the applied potential fills with conducting electrons, and these electrons screen the rest of the material from the potential, preventing conduction anywhere else. In short, a semiconductor with a potential applied is a good model for a 2DEG: It generates a layer of conducting (i.e. freely moving) electrons, which are constrained to a 2D surface.

In the MOSFET used in this lab, the gate is a rectangular sheet. It is separated from the semiconductor, a p-type silicon substrate, by a dielectric. When a gate potential is applied, the energy required for electrons to conduct in the layer of the substrate nearest to the gate is reduced. This is known as "band bending" and is illustrated in Figure 4. At a threshold potential V_T , the bending of the conduction band is large enough that the energy required to conduct falls below the Fermi energy of the material. At this potential, electrons can conduct in the layer closest to the gate.

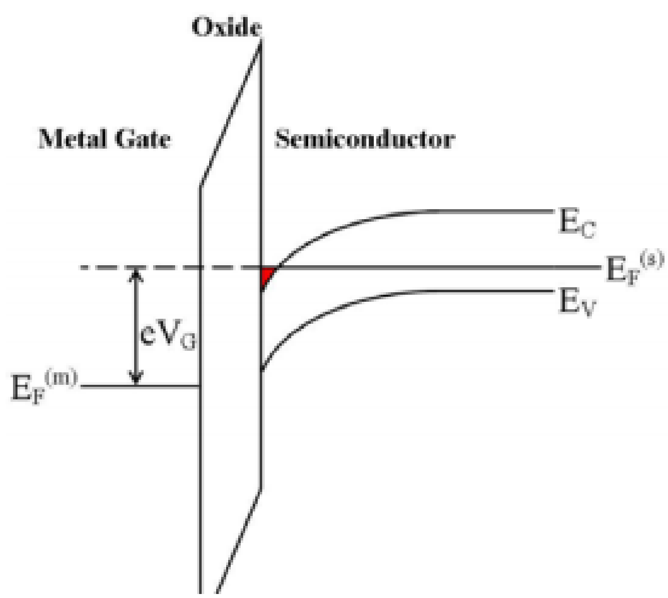


Figure 4: When a gate potential is applied, the energy required for electrons is greatly reduced in the vicinity of the potential. The conduction band is "bent" downwards near the source of the potential. Here, E_F is the material's Fermi energy, E_V the valence energy and E_C the conduction energy. V_G is the gate potential, and eV_G is the amount that the electron energy bands are bent in the layer closest to the gate.

The bending of the conduction band creates two regions in the semiconductor. The layer where electrons conduct (the 2DEG) is called the "inversion layer". In a p-type semiconductor, it is typically positive "holes" that conduct rather than the electrons themselves. An "inversion layer" in a semiconductor is a layer of the material where the majority charge carrier has a different sign than in the rest of the material, as it does in the region where electrons conduct. The positive potential from the gate also creates a "depletion layer" near the gate, an area in which no positive charge carriers are present due to the gate's proximity. Since the MOSFET's source and drain terminals are near the gate, they are located in the inversion layer and the depletion layer. As a result, positive charge carriers cannot access the source and drain terminals and cannot conduct through the semiconductor. Current through the MOSFET is carried completely by electrons in the inversion layer. To learn more about semiconductor physics, see chapter 13 of Kittel & Kroemer.

2DEG Properties

An easy property of the 2DEG to measure is its resistance *R*, or equivalently its conductance *G = 1/R*. (Since the 2DEG consists of all conducting electrons in the semiconductor, the resistance of the semiconductor is equivalent to the resistance of the 2DEG.) From R, the conductivity σ is given

$$\sigma = \frac{L}{WR} \qquad (1)$$

where *L* is the semiconductor's length from source to drain and *W* is its width.

Next we must consider what happens to electrons when they conduct. In free space, electrons accelerate continuously under the influence of an electric field. However, this does not happen in conducting materials: If it did, applying a potential to a circuit would cause current to increase without bound. In practice, interactions between the electrons and the material put a strict bound on the current. Electrons decelerate due to these interactions and their extra energy is released as heat.

To deal with these complex interactions, a simple model is used. First, electrons are given an effective mass m^* to approximate the fact that it takes a different amount of energy to accelerate an electron in a lattice than in free space. Electrons are allowed to accelerate continuously for an average time τ, called the "mean free time". Between these free accelerations, a single collision event occurs between the electron and the lattice, which slows the electron down. The result of these interactions is that an applied electric field *E* results in an average "drift velocity" v_d for electrons, yielding a constant current. The ratio of applied field to drift velocity is called the "mobility" μ:

$$\mu = v_d/E \qquad (2)$$

Based on the mean free time model, the mobility can also be written

$$\mu = e\tau/m^* \qquad (3)$$

The value of m^* in the 2DEG is about $0.19m_e$ in the x and y directions. For more on this model see Brennan chapter 2. In order to determine μ, it is helpful to determine N, the number of electrons in the conduction band per square meter. Since the 2DEG is made up of these electrons, it is equivalently the number of electrons per square meter in the 2DEG. The current (per area) J for gate potential V_G can then be written

$$J(V_G) = eN(V_G)v_\mathrm{d}(V_G) \tag{4}$$

Ohm's law tells us $\sigma = J/E$, which yields

$$\sigma(V_G) = e\mathrm{N}(\mathrm{V_G})\mu(V_G) \tag{5}$$

We can determine N by noting that the MOSFET is arranged like a parallel plate capacitor. A potential (the gate potential) is applied across a dielectric (the silicon oxide insulator), and electric charge builds up in response to this potential. Using standard formulas, the capacitance of the MOSFET can be written

$$C = \frac{\epsilon_{ox}}{t_{ox}} A_{2DEG} \tag{6}$$

where ε_{ox} is the dielectric constant of the oxide layer and t_{OX} is its thickness.

The MOSFET differs from a capacitor because the silicon substrate does not always act like a conductor. For a conductor, any applied potential will result in a collection of charge to nullify that potential. However, in the case of a semiconductor, electrons must overcome an energy gap to enter the conduction band and cannot nullify the applied potential until the gap is overcome. The number of electrons per unit area N in the 2DEG is proportional to the number of electrons whose energies are large enough to overcome this gap:

$$N \propto E_F - E_0 \tag{7}$$

where E_0 is the bottom energy of the conduction band.

This energy gap is first overcome at potential V_T. Thus, for charge in the 2DEG,

$$Q = \frac{\epsilon_{ox}}{t_{ox}} A_{2DEG}(V_G - V_T) \tag{8}$$

Using $N = Q/(e * A_{2DEG})$, this can be related to N:

$$N = \frac{\epsilon_{ox}}{e\, t_{ox}}(V_G - V_T) \tag{9}$$

Finally, using equations (5) and (9), conductivity and gate potential can be related through *N*:

$$\frac{\sigma(V_G)}{e\mu(V_G)} = \frac{\epsilon_{ox}}{et_{ox}}(V_G - V_T) \tag{10}$$

If $\sigma(V_G)$ is known, equation (10) gives us a way to calculate $\mu(V_G)$, unlocking other properties of the 2DEG. However, it requires that we know ε_{ox}, t_{ox} and V_T. ε_{ox} is known to be about 3.9, but t_{ox} and V_T must be determined in the lab for a particular MOSFET. The next section will introduce the theory needed to determine these values.

Landau Levels

Since electrons in a 2DEG can be treated as free particles, their energy states are not quantized: They can take on any momenta p_x and p_y. That changes when a magnetic field is applied perpendicular to the 2DEG. When this happens, electrons form orbits around the magnetic field lines, and the wave functions of these orbits are quantized. The orbits are called "cyclotron orbits", and the allowed orbital energy levels are called "Landau levels".

Using arguments about the behavior of charged particles in magnetic fields (see Landau and Lifshitz [3] chapter 15), it can be shown that the Landau levels for a given magnetic field are equivalent to the energy levels of a quantum harmonic oscillator. The frequency of the equivalent harmonic oscillator is called the "cyclotron frequency" ω_c:

$$\omega_c = \frac{eB}{m*} \tag{11}$$

where *B* is the magnetic field strength. Landau levels can then be written as the energy levels of this quantum oscillator. Let E_0 be the bottom energy of the conduction band. Then the n^{th} Landau level is located at

$$E_n = E_0 + \left(n + \frac{1}{2}\right)\hbar\omega_c \quad (n \geq 0) \tag{12}$$

Electrons fill these energy states by "condensing" into groups, filling the nearest available Landau level. This is illustrated in Figures 5A & 5B.
It is possible to use Landau levels to obtain information about the 2DEG. To do this, we must first analyze the 2DEG's density of states function.

The density of states function *g(E)* measures the total number of energy states available to electrons. For simplicity, this manual will take *g(E)* to be the density of states per unit area, in units (energy states)/(energy * area).

Since electrons in the 2DEG can be approximated as free particles, the 2DEG's density of states function is the same as that for an ideal gas. Schroeder [1] section 6.7 uses geometric arguments to determine *g(E)* for a 3D ideal gas. Using the same method, it is straightforward to derive the result for a 2D gas. Let E_0 be the bottom energy of the conduction band, and let $B = 0$. Then

$$g(E) = \begin{cases} \dfrac{m^*}{\pi\hbar^2} : E > E_0 \\ 0 : E < E_0 \end{cases} \tag{13}$$

When B is nonzero, only energy levels located at Landau levels can be occupied. Therefore, *g(E)* in this case can be approximated as a series of delta functions located at the Landau levels. (In reality, due to lattice impurities and nonzero temperature, *g(E)* is not made of sharp delta functions: it's made of Gaussians centered on each Landau level. However, approximating these to be delta functions is useful and close to accurate.)

Figure 5A: A typical electron density of states function when no magnetic field is applied. E_0 is the bottom of the conduction band, and there are no available energy states between E_0 and the top of the valence band. In the conduction band, there are g_0 states available per unit energy

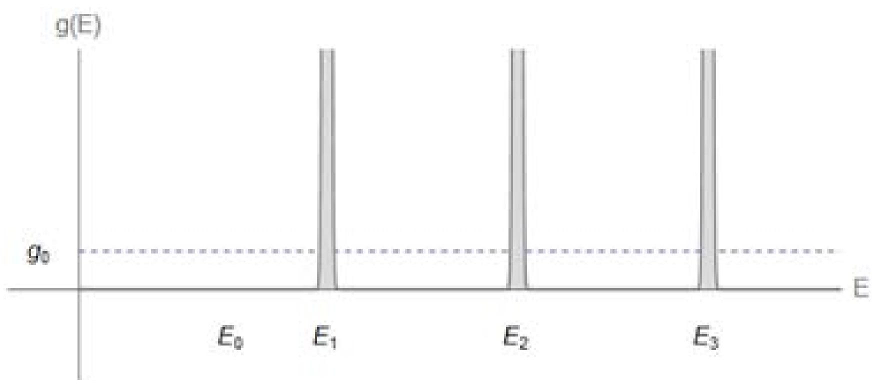


Figure 5B: The density of states function in a magnetic field. Electrons can no longer occupy all energies in the conduction band. Instead, they group into regularly spaced Landau levels at energies E_1, E_2, E_3, etc. In this case, temperature and impurities in the lattice are quite low and electrons are sharply concentrated around Landau levels.

The density of states function is complimented by the Fermi function *f(E)*, which yields probability that a state at energy E will be occupied if it is available. For Fermi energy E_F,

$$f(E) = 1/(1 + e^{(E-E_F)/kT}) \tag{14}$$

Combining these gives the expected number of electrons per energy state, called the occupation of states *n(E)*. This is given by

$$n(E) = g(E)f(E) \tag{15}$$

Like *g(E)*, it is given here per unit area. When integrated, *n(E)* yields the total number of electrons per unit area (in all energy levels) in the 2DEG. This is equivalent to N, yielding the equation

$$N = \int_E g(E)f(E)dE \tag{16}$$

This integral is difficult to evaluate in general. However, some simplifying approximations can be made. For very low temperatures we can estimate the Fermi function *f(E)* by

$$f(E) \approx \begin{cases} 1 : E < E_F \\ 0 : E > E_F \end{cases} \tag{17}$$

Thus when $T \approx 0$, N is the sum of all electrons between E_0 (the start of the conduction band) and E_F (the bound imposed by the Fermi function). This yields

$$\begin{aligned} N &\approx \frac{m^*}{\pi\hbar^2}(E_F - E_0) \\ &= \frac{\epsilon_{ox}}{et_{ox}}(V_G - V_T) \end{aligned} \tag{18}$$

The second equality is from equation (9) relating N *to* V_G. Equation (18) therefore relates V_G to the maximum energy allowed in the 2DEG, E_F:

$$V_G(E_F) = V_T + \frac{m^* e t_{ox}}{\pi\hbar^2\epsilon_{ox}}(E_F - E_0) \tag{19}$$

Note that this applies to calibration only when $B = 0$ and the density of states is flat. When $B > 0$, Landau levels form at energies ω_C apart. Since electron energies are no longer uniformly distributed, N is no longer proportional to V_G and equation (9) breaks down.

Instead, N is limited by the number of Landau levels between E_0 and E_F.

What can we do with this information? Recall from equation (5) that σ is proportional to N. This equation is of little use by itself because σ also depends on μ and μ*'s* value is still unknown. However, this problem can be sidestepped by plotting σ vs. V_G when Landau levels are present. At values of V_G corresponding to Landau levels, N will increase very suddenly as a new Landau level enters the conduction band, and this increase will correspond to an increase in σ which we can measure directly. The pattern of sudden increases and stagnation give this lab the name "quantum oscillations", where the oscillations occur in N over changes in V_G. The locations of these oscillations can be matched with the known values for the Landau levels using equation (19). Finally, we can use the fact that equation (19) depends on V_T and t_{ox} to determine these values.

Preliminary questions

1. Use equation (19) to determine the locations of the Landau levels in terms of the gate potential.
2. For a semiconductor, let energy E_V mark the top of the valence band and E_C mark the bottom of the conduction band. Assume the density of states function in each band is given by the same constant value. Finally, assume that there exactly enough electrons to fill the valence band. Show that the Fermi energy is located exactly in the middle at $(E_C + E_V)/2$. (If you are unfamiliar with Fermi energy, Kittel and Kroemer chapter 3 is a good reference. Additionally, this question is very similar to Schroeder problem 7.33.)
3. What behavior would occur if you applied a negative potential to the MOSFET instead of a positive one?

Equipment

The equipment consists of a MOSFET, a lock-in amplifier, a function generator, 2 small DC power supplies, isolation transformers, a multimeter and a superconducting magnet in a LHe Dewar w/ a control system and a large DC power supply. Information on the MOSFET, lock-in amplifier and superconducting magnet system follow:

MOSFET

The MOSFET is very sensitive to static electricity – be sure to ground yourself before handling the MOSFET or you may inadvertently destroy it. Student pairs are no longer asked to install a new MOSFET at the start of their experiment. However, should you need to change the MOSFET or are just curious about the MOSFET and its handling procedure, see appendix B

Lock-in Amplifier

The "lock-in amplifier" is basically an AC voltmeter that can lock onto a selected frequency and amplify that frequency preferentially - other parts of the signal are filtered out. The output of the lock-in can be read from an analog scale or digital display on the lock-in's front panel. However, we will feed the lock-ins +/-10V output signal to a scope where we will display and collect data. The lock-in's +/-10V output is scaled to the sensitivity setting on the lock-in. For example, if the lock-in is set on 10mV scale and measuring a 10mV input signal (full scale deflection), the lock-in output signal will be 10V. However, if the lock-in is set on the 20mV scale and measuring a 10mV signal (a ½ scale defection), the lock-in output will only be 5V. For more information on the lock-in amplifier see Appendix A and the lab wiki.

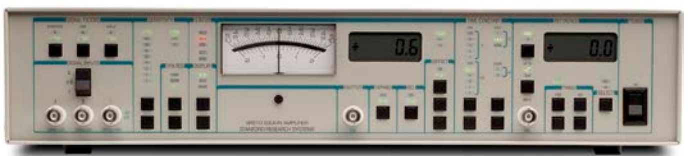


Figure 6: The SR510 lock-in amplifier

Superconducting Magnet

The Croymagnetics superconducting magnet system consist of a 6 Tesla superconducting magnet in a cryogenic Dewar with associated electronics. The magnet must be at LHe temperatures to operate. If it is operated above LHe temperatures, it will likely be destroyed and could be dangerous to anyone in the lab. The cryogenic Dewar holds 45 cm of LHe. The minimum LHe level for safe operation of the magnet is 12.5 cm.

The Dewar holds ~4 liters of LHe, the typical boil off rate is ~0.4 liters/hour and there must be ~1 liter in the Dewar to cover the magnet. So, you could have 7.5 hrs to use the magnet. However, the lower the LHe level, the faster it boils off. Occasionally an ice bridge forms in the Dewar and the LHe boils off much faster than usual. It is important to monitor the LHe level when working with the magnet.

The magnet system has five parts (See Figures 7 & 8):

1) Model 12 LHe Level indicator – displays the LHe level.
2) Superconducting magnet inside a cryogenic Dewar – The Dewar is a vapor shielded liquid helium cryostat. Staff member supervision is required when filling this Dewar with cryogens. Please follow the provided instructions carefully. Before you may handle LN2 you will need to receive LN2 safety training from the TA.
3) HP6259B DC power supply – provides current to the magnet
4) Model 60 Programmer/Monitor – controls the rate of current flow to the magnet.

5) Model 30 Persistent Switch – connects/disconnects the power supply and magnet. Caution: Before reconnecting the power to the magnet, the current setting on the controller must match the current value when the power was disconnected. If the current values do not match the magnet will likely quench. This will result in a dramatic and potentially dangerous boil off of all remaining LHe in a matter of seconds. This could seriously damage the magnet and Dewar system.

Figure 7: The Model 30 Persistent Switch, Model 12 LHe Level indicator and Model 60 Programmer/Monitor power supply controller

Figure 8: The HP6259B DC power supply

Experimental Overview

In this lab, the conductance and transconductance of a p-type silicon substrate of a MOSFET will be measured at three different temperatures: room, LN2 and LHe. While at LHe temperature the transconductance will be measured at several magnetic fields.

The conductance circuit – Figure 9 - relates the conductance *(Ω^{-1})* of the semiconductor to the gate voltage *(V_G)*. The Sin Out from the lock-in provides the Drain - Source voltage as well as the modulation required for the lock-in. To prevent grounding issues, the signal is passed through an isolation transformer. The resulting current flow is dependent on the conductance of the substraight which is controlled by V_G. The voltage associated with this current flow is fed into the lock-in via a second isolation transformer. Note: the 2 isolation transformers are interchangeable. A typical oscilloscope display for this circuit is shown in Figure 10.

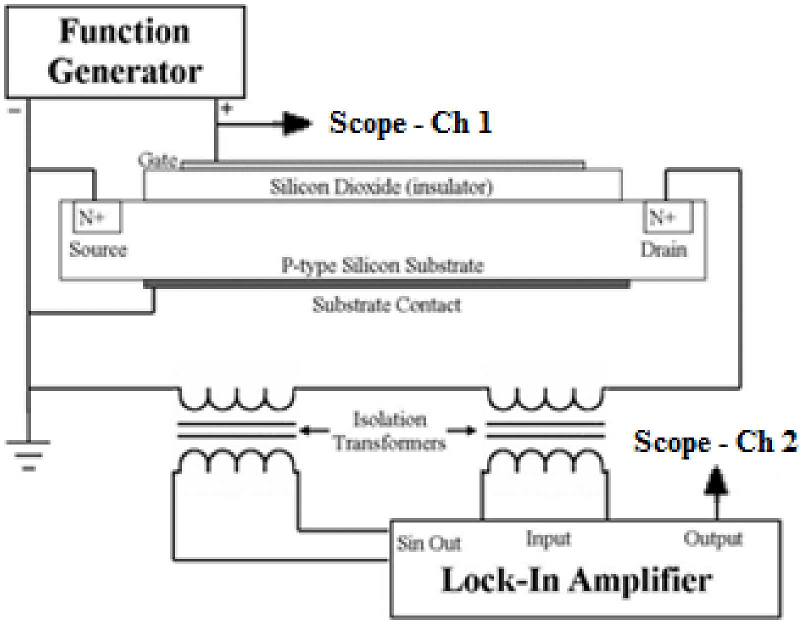


Figure 9: Circuit for measuring conductance of a p-type silicon substrate.

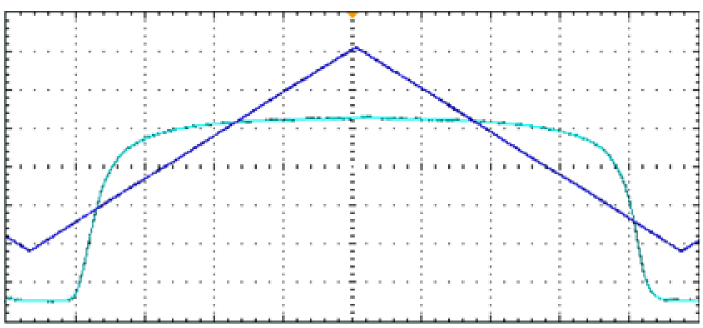


Figure 10: A typical conductance curve as displayed on an oscilloscope. The purple line represents the Gate voltage; the green line represents the Drain - Source voltage.

The transconductance circuit – Figure 11 - relates the derivative of the conductance *(Ω^{-1})* of the semiconductor to the gate voltage *(V_G).* $\frac{\Delta\sigma}{\Delta V_G} \approx \frac{\partial\sigma}{\partial V_G}$

In this circuit the Drain - Source voltage is provided by a small DC power supply. Sin Out from the lock-in provides modulation to V_G; the modulation of V_G causes modulation of the Drain - Source current which ultimately shows up on the input to the lock-in. It is this change in modulation coupling that cause the lock-in output to be the derivative. A typical oscilloscope display for this circuit is shown in Figure 12. Note the red line intercept labeled V_T, one definition of the threshold voltage, V_T, is the intercept of the rapidly rising, almost linear, section of the V_G curve with 0V.

If the mobility μ were constant, the graph would simply be a straight line rising from the V_G axis, reflecting the linearly increasing N. The largest features are easily explained. The intersection of the curve with the V_G axis at V_T is not abrupt because lattice defects and oxide impurities interact with the electrons, causing the collision rate 1/τ to be relatively large. As more electrons are added, the collision rate per electron approaches a lower, more constant value due to screening, at which point the curve approaches the linear form predicted by our theory. At still higher voltages the well in the z-direction becomes steeper, increasing confinement, causing the electrons to collide more often with any surface defects. Thus the conductivity tapers off at high gate voltages.

Ultimately this experiment must be done at LHe temperatures so that $\omega_C\tau \geq 1$ for an electron in the MOSFET inversion layer; in other words, so that scattering does not broaden the Landau levels too much.

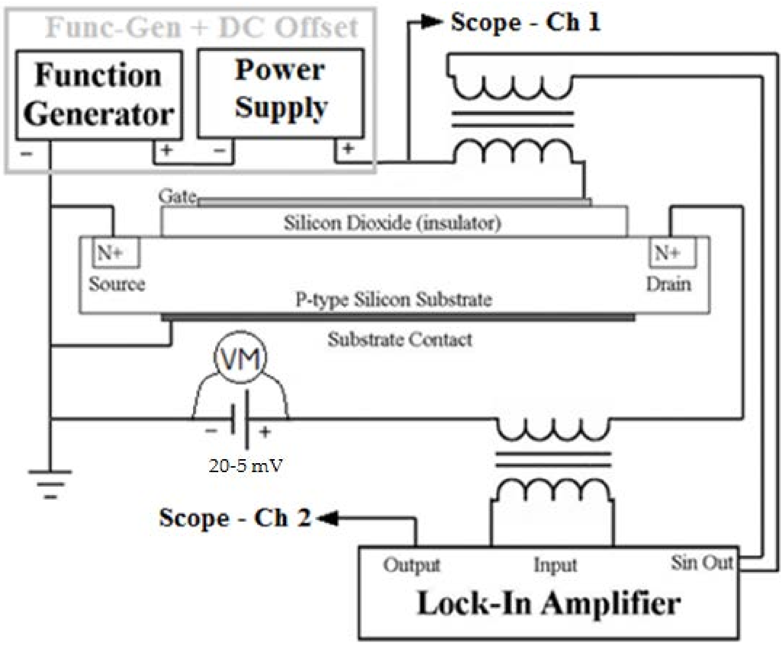


Figure 11: Circuit for measuring transconductance

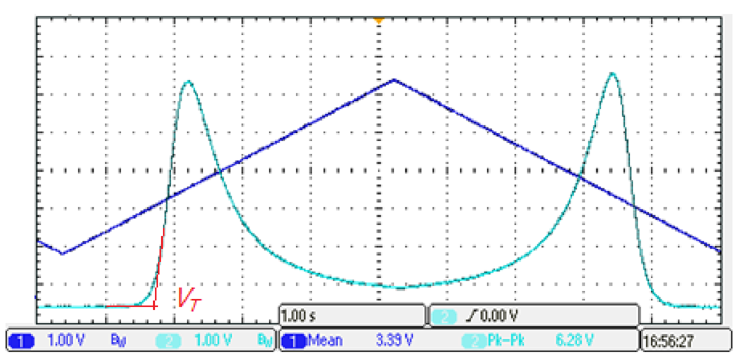


Figure 12: A typical transconductance curve as displayed on an oscilloscope. The purple line represents the Gate voltage; the green line represents the derivative of the Drain - Source voltage.

Experimental Procedure

1) Construct the conductance circuit shown in Figure 9. Adjust the various parameters to get an oscilloscope display similar to Figure 10. The function generator frequency will need to be quite low. Try a 0.1 Hz, 6V triangle wave as a starting point. You may need a lower frequency to get optimal data. On the lock-in you will need to determine a reasonable sensitivity, and a modulation frequency and optimal phase. Ideally the lock-in will be set such that the signal maximum gives a near full scale deflection. Remember to record your frequency, phase and sensitivity. Once you have a reasonable signal, collect room temp data. You can collect data on the Tektronix oscilloscope however, you will get much nicer data if you use the Analog Discovery digital oscilloscope attached to the computer.

2) Construct the transconductance circuit shown in Figure 11. Adjust the various parameters to get an oscilloscope display similar to Figure 12. Note: The lock-in settings might need to be changed. The Drain – Source voltage will have provided by a small DC power supply. The appropriate voltage is 20-5mV. Once you have a reasonable signal, collect room temp data.

3) The rest of the measurements will be taken at cryogenic temperatures. The LN2 data can be taken during the LHe cool down. Before you can work with LN2 or LHe you will need to receive cryogenic safety training – please consult a TA.

4) Request an LHe transfer at least 1 day in advance. LHe is very expensive and the transfer process is rather involved. Before LHe will be transferred, you will need to:
    a. Demonstrate your proficiency at changing the wiring and settings from a good conductance display to a good trasconductance display and back to a good conductance display. Note: Once the MOSFET cools down, its characteristics will change dramatically, be sure you are proficient enough with the electronics to make the required adjustments efficiently.
    b. Demonstrate that you know how to operate the superconducting magnet system.
    c. Have recorded the frequencies, amplitudes, offsets, phases, voltages etc. you will be using during the experiment.
    d. Create a data taking table in your notebook that includes the proposed magnet current and other parameters at which you intent to take data.
    e. Identify a 4-hour block of time during which both students will be available to work on the experiment without interruption. The LHe transfer process will likely take 1 hr and you should allow 3 hours to take data. The data collection will likely not take that long but you have to

allow time for things to go wrong; it is not uncommon for students to collect their all of their data before they discover they did something wrong. They sometimes have to retake all of their data – it happens.

5) The cryogenic Dewar has to be precooled with LN2 for about ½ hour. Once the LN2 is in the Dewar, you can take LN2 temperature data.

6) Once the LHe is transferred, you should begin taking data promptly. The LHe should last about eight hours but don't be lulled into a false sense of abundance. Things do go wrong and addressing problems can be a big time sink. And, remember, the evaporation rate increases as the LHe level decreases.

7) Take conductance and transconductance data at LHe temperatures. The transconductance settings may be particularly different.

8) Read and follow the *Controlling the Superconducting Magnet* instructions below and collect transconductance data for at least 4 different magnetic fields that display oscillations. The more oscillations per data set the better.

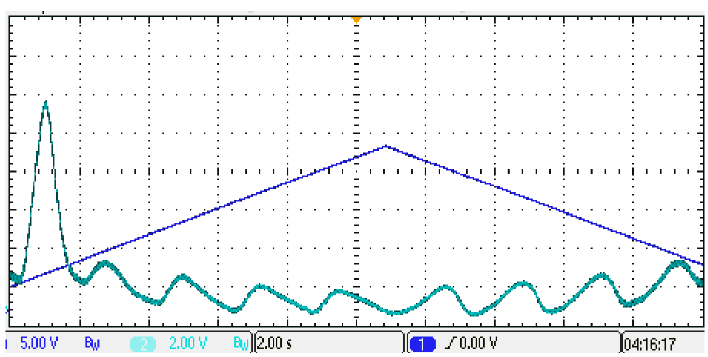


Figure 13: Quantum Oscillations at a high magnetic field as displayed on an oscilloscope. The purple line represents the Gate voltage; the green line represents the derivative of the Drain - Source voltage.

9) The MOSFET should be located in the center of the magnet's vertical axis. A piece of yellow tape near the top of the probe shaft and near the bottom on the outside of the Dewar mark this depth. To confirm the magnet is working, place a piece of iron – such as a wrench - on the outside of the Dewar at the yellow tape. Be careful, if the magnet current is high, it will pull the wrench out of you hand. If you have any question about controlling the magnet, please ask.

10) Once you have confirmed all of your data was correctly collected, you can move on to collecting calibration data.

<u>Controlling the Superconducting Magnet</u>

Please read all of these instructions before you start controlling the magnet. It is important that the following steps are carried out in the order presented - failure to do so could damage the equipment or even be dangerous. As mentioned above, operating the superconducting magnet with less than 12.5 cm in the magnet Dewar is dangerous. To be safe, please stop all current flowing through the magnet if the LHe level is below 14 cm. Note: The Pause position on the Ramp Control does not work so these instructions differ from those in the manufactures manual. Also Note: The Display selector on Model 60 controller is slightly broken, it will turn to a position CCW of P.S. Current. This position does nothing; please do not set the selector in this position.

a) Turn the on the Model 60 Programmer/Monitor and set the Ramp Control to Down.

b) Turn on the HP DC Power Supply; allow the power supply to warm up for about 5 minutes.

c) On the Model 60, set the Current Monitor, Display to Ramp Rate and confirm the rate ~ -0.7. Note: The Ramp Rate control is locked and should not be adjusted. Set the Display to Current Limit and confirm the limit = 0.

d) Confirm that the LHe level is well above 14 cm and rotate the key on the Model 30 Persistent Switch 90^o CW to Connect and wait 1 min.

e) With the Current Monitor set to Current Limit, set the limit to your first desired current – it is best if you collect data from lower currents to higher currents.

f) Set the Ramp Control to Up. When the Current Monitor LED lights, you have reached your desired current.

g) Record the value of this current and rotate the key on the Model 30 Persistent Switch 90^o CCW to Disconnect and wait 1 min.

h) Set the Model 60 Ramp Control to Down, once the power supply current is 0, you may collect data. To confirm that the current is 0, set the Current Monitor, Display to PS Current.

i) To go to a higher current, Set the Model 60 Ramp Control to UP and wait until Current Monitor LED lights. Confirm that the current displayed matches the current you recorded in step g). Rotate the key on the Model 30 Persistent Switch 90^o CW to Connect and wait 1 min.

j) Set the Current Limit to your next desired current. When the Current Monitor LED lights, you have reached your desired current.

k) Repeat Steps g) - j) until you have collected all of your data. Note, the current used in the step i) should always be the current recorded the last time the Model 30 Persistent Switch was set to Disconnect.

l) To go to a lower current, use step i) to set Model 30 Persistent Switch to Connect. Set the Model 60 Ramp Control to Down and wait until the power supply current is 0. Set the Current Limit to the desired value. Follow steps g) – h).

m) When you are done with the magnet, use step i) to set Model 30 Persistent Switch to Connect. Set the Model 60 Ramp Control to Down and wait until the power supply current is 0. Set the Current Monitor to PS Current and confirm the current is 0. Set Model 30 Persistent Switch 90^o CCW to Disconnect and wait 1 min.

n) You may now turn off all of the magnet control electronics.

Calibration

As mentioned above, the output of the lock-in goes from -10V to +10V and is scaled to the sensitivity setting. Therefore, the voltage of the lock-in is not the voltage of the measured signal - see *Equipment / Lock-in Amplifier* above, *Appendix A* and/or the lab wiki for more information. Also, as the conductance of the p-type silicon substrate changes, the optimal phase of the lock-in changes. To make sense of your data, you will need to go through this calibration process.

1) Replace the MOSFET's Drain & Source with a variable resistor and construct the conductance circuit - minus all Gate and Substrate connections. See Figure 14

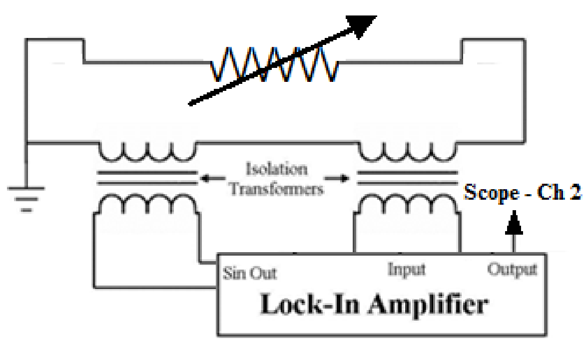


Figure 14: Calibration Circuit

2) Look through your data and determine the range of resistances you measured.
3) Determine a selection of resistances to measure.
4) Set the lock-in to the frequency, phase and sensitivity you used to record your data.
5) Adjust the variable resistor (decade box) accordingly to measure the calibration circuit's output for each of your resistors. Record your data.
6) Since this calibration is about the lock-in's response to conductance changes in the MOSFET and how these changes are represented on the oscilloscope, it is not necessary to separately calibrate the transconductance circuit.
7) Remember to collect the specifications of the MOSFET and the dimensions of the silicon chip in the MOSET – on the wall above the Dewar.

Analysis

Examine your $B \neq 0$ transconductance curves. The minima of the oscillations identify the Landau levels. The Landau levels should occur at regular intervals relative to V_G. See Figure 15 for clarification.

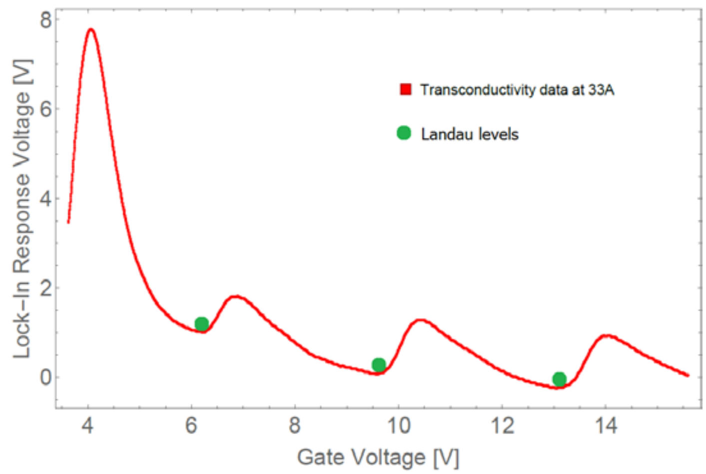


Figure 15: A transconductance curve of a MOSFET in a magnetic field. Notice the oscillations, the local minima – green dots – identify the Landau levels. Note: Ideally there would be more several more minima.

Plot the gate voltage of each Landau level against its index n then plot a line of best fit through the points from each magnetic field, as in figure 16. These lines should converge to a common y-value for $n = -1/2$; this is the threshold voltage V_T .

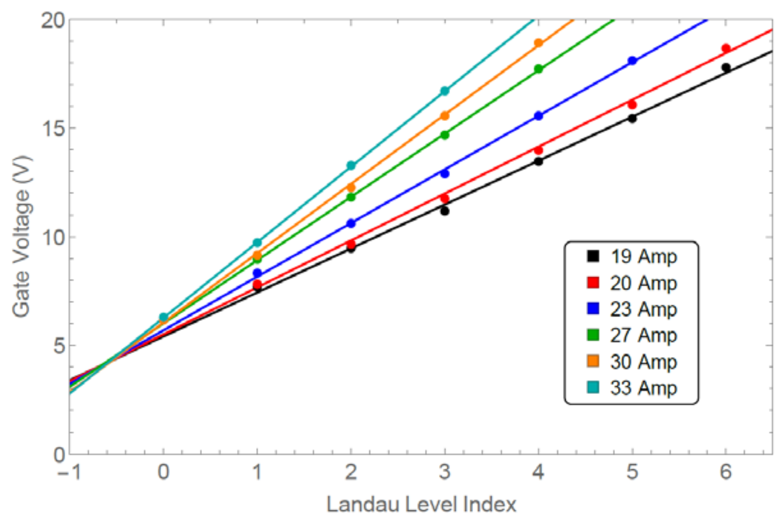


Figure 16: A plot of Landau level number versus gate voltage for several magnetic fields. You will need to identify the index of the Landau levels from context.

In addition, equation (19) relates V_G to electron energies and equation (12) yields the energies of the Landau levels. You can determine t_{ox} (the oxide layer's thickness) from these equations and the slopes of the lines you plotted. Determine t_{ox} and V_T. Given these values, determine N (equation 9).

Next, examine your conductance circuit calibration data. Determine a relationship between the voltage at the oscilloscope V_{OM} and the resistance experienced by the MOSFET.

From this resistance and the dimensions of the MOSFET, you can find the MOSFET's conductivity (equation 1). Use each of the conductance curves that you took and determine the corresponding "conductivity curves." That is, determine $\sigma(V_G)$ for each temperature (room, liquid nitrogen and liquid helium).

Finally, there is now enough data to determine the mobility μ and mean free time τ. Like conductivity, these are functions of temperature and of V_G. Determine the $\tau(V_G)$ and $\mu(V_G)$ curves from $\sigma(V_G)$ for at least one temperature. In your analysis, discuss the shape of these curves. What do they say about how the material behaves under different gate voltages? Do they agree with the expectations outlined in this lab?

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Appendix A: (Lock-in Amplification)

A lock-in amplifier is basically a voltmeter that employs sophisticated filtering techniques to identify and selectively amplify a very weak signal from significant background noise. The technique used in the lock-in is called phase-sensitive detection, otherwise known as lock-in amplification. This technique uses a reference frequency at a particular phase which is injected into the experiment and becomes a part of the experiment's output signal. Amongst all of the noise in the experiments output, the signal we are looking for will be modulated at the frequency of the reference signal; only the signal we want will be modulated, none of the noise will be modulated. The experiment output signal is fed into the lock-in, and through a process called mixing, the lock-in isolates the modulated signal from the noise.

In the mixing process, the output signal from the experiment and the original reference signal are *mixed* such that the combined waveform is the product of the two initial waveforms. If the two signals have identical frequency, the product frequency will be twice the frequency of the two original signals. This product signal is amplified and, since noise does not have a regular frequency, the noise is not amplified.

If the two signals that are mixed are in phase and have the same amplitude, the p-p voltage of the product-wave goes from 0 to 2X volts instead of from –X to +X volts as the two initial signals did. If the two signals are 180^o out of phase, the p-p voltage of the product-wave goes from 0 to -2X. And, if the two signals are 90^o put of phase, the p-p voltage of the product-wave goes from -X to X.

The lock-in rectifies the product-wave and the value of this DC voltage is the output of the lock-in. So, when the output from the experiment and the original reference signal are in phase, the rectified signal = X volts DC; when these two signals are 180^o out of phase, the rectified signal = -X volts DC; and when these two signals are 90^o out of phase, the rectified signal = 0 volts DC. For a more formal description of this process, see the following:

If the experiment output and the reference signal are given by

$$V_{ref} = V_r \sin(\omega_r t + \theta_r)$$
$$V_{sig} = V_s \sin(\omega_s t + \theta_s),$$

Then the combined signal is given by

$$V_t = V_r V_s \sin(\omega_r t + \theta_r) \sin(\omega_s t + \theta_s)$$

$$V_t = \tfrac{1}{2} V_r V_s \{\cos[(\omega_r - \omega_s)\, t + \theta_s - \theta_r] - \cos[(\omega_r + \omega_s)\, t + \theta_s + \theta_r]\}$$

Clearly, when $\omega_s = \omega_r$ (experiment signal has the same frequency as the reference), the resulting signal will be a wave with twice the reference frequency ($2\omega_r$) and some offset determined by the phase difference in reference and return signals, $\cos(\theta_s - \theta_r)$. The combined signal is sent through a low-pass filter with a bandwidth $\Delta f = \frac{1}{4T}$, where T is the time constant of the lock-in. In the ideal case of infinite T, only the DC portion of the signal with $\omega_s = \omega_r$ is retained (no time dependence at all), and the measured voltage will be a DC signal given by

$$V_{measured} = \frac{1}{2} V_r V_s \cos(\theta_s - \theta_r)$$

Therefore, the magnitude of the measured DC signal is proportional to the amplitude of the desired signal and the cosine of the phase difference between the experiment and reference signals. To maximize the measured signal, the phase difference must be minimized. To do this, you will utilize the function on the lock-in that allows you to change the phase of the reference signal. Take care to note that a phase difference of 90° causes the signal to vanish, while a phase difference of 180° gives a negative signal.

It's worth noting that the whole point of lock-in amplification is to remove background signal that is not at the reference frequency. It's obvious that in the equation above, if $\omega_s \neq \omega_r$, there will be no DC offset, the signal will just be two waves added together. The low-pass filter after the mixer ideally would remove all periodic signals, leaving just the DC offset when $\omega_s = \omega_r$ – the desired signal. Of course, the low-pass filter is not ideal and cannot remove all the background noise – waves where ω_r and ω_s are very close will have very low frequency signals that will get through the low-pass filter. Frequencies within a bandwidth (defined above) of the reference frequency, *i.e.* $\omega_r - \Delta f < \omega_s < \omega_r + \Delta f$, will appear in the final measurement.

The time constant of the lock-in used in this experiment can go as high as 100 seconds, giving the low pass filter a bandwidth of 0.0025 Hz. However, increasing the time constant comes at the cost of losing sensitivity to rapid changes in the signal.

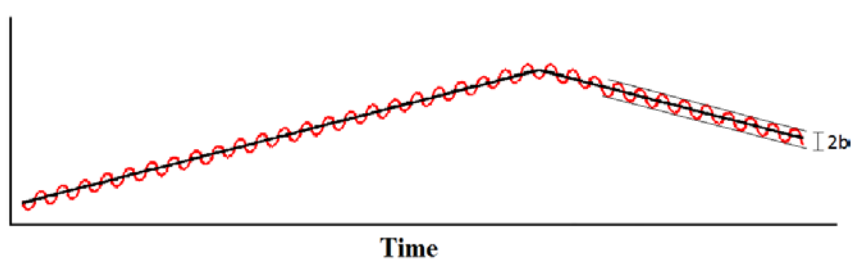

Figure 17: Effect of modulation on input signal: Black line represents the triangle wave applied to the Gate via the Func-Gen + DC Offset. Red sine wave represents small sinusoidal modulation of the triangle wave caused by the reference signal. These variations in the triangle wave will be present in the experiments output signal. Note: Not drawn to scale.

The reference signal is used to introduce a small sinusoidal oscillation in the extremely low frequency triangle wave being applied to the Gate. This extremely low-frequency triangle wave varies the electric field between the Gate and Substraight through some range. However, since $\omega_{ref} \gg \omega_{ramp}$ we can treat the overall signal as a DC signal which varies slowly with time plus a small oscillating perturbation. Since the return signal depends on the total input signal, we write the amplitude *A* of the return signal as

$$A = A(V_t + \Delta V_t)$$

where V_t is the main input signal and $\Delta V_t = V_1 cos(\omega_r t)$ is the perturbation provided by the amplifier reference overlay. If we Taylor expand *A* around V_t, we find

$$A(V_t + \Delta V_t) = A(V_t) + \frac{dA}{dV_t}\Delta V_t + \frac{1}{2}\frac{d^2A}{dV_t^{\,2}}(\Delta V_t)^2 + \cdots$$

$$A(V_t + \Delta V_t) = A(V_t) + \frac{dA}{dV_t}V_1\cos(\omega_r t) + \frac{1}{4}\frac{d^2A}{dV_t^{\,2}}V_1^{\,2}(1 + \cos(2\omega_r t)) + \cdots$$

Since the signal $A(V_t + \Delta V_t(\omega_r))$ passes through a lock-in amplifier, only the part of the signal which is periodic with frequency ω_r emerges in the final measurements ($\cos(\omega_r t) = \sin\left(\omega_r t + {}^{\pi}/_{2}\right)$). As such, the signal you will observe on the oscilloscope is the derivative of the expected signal and not the resonance signal itself.

See Figure 7 for a conceptual picture of why the lock-in yields the derivative and not the expected signal. As you sweep the voltage through a resonance, the reference signal moves back and forth with amplitude V_1 as described above ($\Delta V_t = V_1 cos(\omega_r t)$). As the total signal oscillates, a portion of the output signal $A(V_t + \Delta V_t)$ oscillates at the same frequency with amplitude determined by the derivative of the expected signal at that value. In essence, the oscillations make the output signal *A* slide up and down the expected signal - the Lorentzian in Figure 18 - at the reference frequency, with an amplitude proportional to the local slope of the curve. This amplitude is denoted as ΔA in the figure and is given by the second term in the Taylor expansion. When the lock-in amplifier isolates the portion of the return signal at the reference frequency, the DC signal registered on the oscilloscope is the amplitude of that oscillation, which is proportional to the derivative of the absorption signal. The phase change in the return signal on opposite sides of the peak tells the lock-in that the slope of the absorption curve has become negative.

For a more information on lock-in amplifiers see Appendix A of Preston & Dietz (p 367) in the B&H 203 reference library, the SR510 Lock-in Amplifier manual under Equipment Manuals on the lab wiki, and SRS App Note 3: About Lock-in Amplifiers and SRS App Note 6: Dynamics Signal Enhancement, both under Technical Tips & References on the lab wiki. https://wiki.brown.edu/confluence/pages/viewpage.action?pageId=1164172

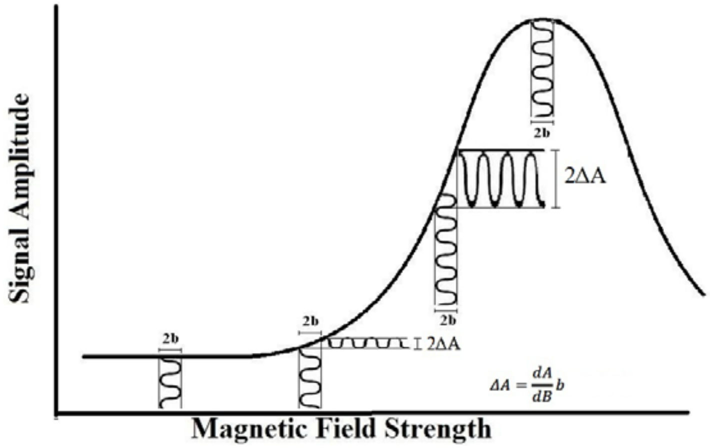

Figure 18: Why the lock-in gives the derivative of the absorption signal. The vertical sine waves in the image represent the signal modulating the triangle wave. Note: their amplitude is constant. The horizontal sine waves represent the amplitude of the resulting signal which is fed into the lock-in. Note: the amplitude of these waves varies with the slope of the curve.

Appendix B: (MOSFET instructions)

Carefully remove the LHe probe from the Dewar and lay it down in a safe and convenient location. Remove the Teflon tape and old MOSFET from the probe. Before handling a new MOSFET, make sure you are not static electrically charged. You can discharge yourself by touching an electrical ground such as the BNC on an oscilloscope. Remove the new MOSFET from its package and insert it into the end of the LHe probe – use Figure 19 as a guide. Secure the MOSFET in the probe, using one 3-4" piece of yellow Teflon tape - See Figure 20. Once the MOSFET is in the probe and properly secured, insert the probe back in the Dewar. There should be yellow tape on the neck of the probe to indicate the proper depth. The MOSFET should end up at the center of the super conducting magnet. A piece of yellow tape on the outside of the Dewar indicates this location.

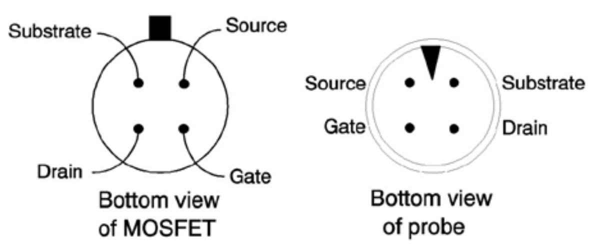

Figure 19: MOSFET and probe pin-out.

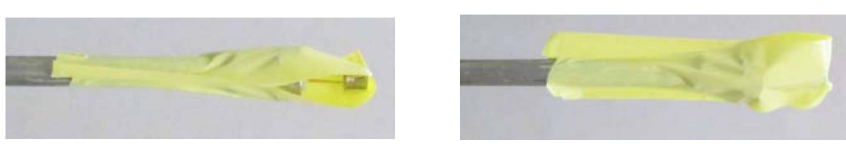

Figure 20: Taped MOSFET